

Chapter notes: 3 Algebraic structures

Overview

Although several sections of this chapter do not map directly onto the syllabus, it focuses on some core skills which are needed across all other chapters. It is recommended that the chapter is covered after chapters 1 and 2, so that sufficiently complex structures are available. We would recommend four hours of teaching time.

Introductory problem

After some time attempting this it should become apparent that none of the analytical techniques known so far can solve this equation. The worked solution is given at the end of the chapter, page 84; the idea being that students should be able to answer the question using the methods covered in the chapter.

3A Solving equations by factorising, p72

An example for the ‘Research explorer’ comment might be $xy - x - y + 1 = 37$.

Hints for the grade 7 questions:

3. Take out a factor of 5^x .
4. How can an expression of the form a^b equal 1?

3B Solving equations by substitution, p73

Hints for the grade 7 questions:

6. Use the change of base formula to get all logarithms to base 2.

3C Features of graphs, p76

It is vital that students are competent at using the graphing functions on their calculator, especially being able to view graphs in appropriate scales and interpret artefacts from the graphs.

Hints for the grade 7 questions:

4. This question deliberately makes the choice of scales difficult. Separate scales will be needed in positive and negative domains.

3D Using a graphical calculator to solve equations, p80

Hints for the grade 7 questions:

3. Sketch the graph of the described situation. Rearrange each new equation into the form $\ln x = \dots$ and consider the new gradient.
4. There are several possible values – you are only required to find one. Try some values on your GDC.

3E Working with identities, p83

An example to show that reduction to a true statement is a logically-flawed proof method might be:

$$1 = 3$$

Subtract 2:

$$-1 = 1$$

Square:

$$1 = 1$$