

Chapter notes: 4 The theory of functions

Overview

This is quite a theoretical chapter. We recommend that functions and ideas from chapters 1 and 2 are taught first, to allow sufficient breadth of functions to be used. It requires approximately five teaching hours.

Introductory problem

This problem is really about domains. It works for any number except zero. The worked solution is given at the end of the chapter, page 109; the idea being that students should be able to answer the question using the methods covered in the chapter.

4A Function notation, p86

There are no specific teacher notes for this section.

4B Domain and range, p88

The proof of the difficulty of setting $\frac{1}{0} = \infty$ (as described in the ‘From another perspective’ box on page 89) is that we also would have to set $\frac{2}{0} = \infty$ so that:

$$\frac{1}{0} = \frac{2}{0}$$

$$\Rightarrow 1 = 2$$

Some academics have indeed suggested that $\frac{1}{0}$ should be accepted, but this requires that division is no longer the inverse operation of multiplication. See the link below for more information:

http://www.bbc.co.uk/berkshire/content/articles/2006/12/06/divide_zero_feature.shtml

The issue explored in the ‘Research explorer’ box (page 91) explores the difficulties with raising negative numbers to rational powers. That is, is $(-2)^1$ the same as $((-2)^{0.5})^2$?

Hints for the grade 7 questions:

7. The difficulty here is in avoiding multiplying $\frac{8x-4}{x-12} > 0$ by $x-12$.

8. Consider the domain without the restriction.

4C Composite functions, p92

Hints for the grade 7 questions:

8. The range of the inner function must be the domain of the outer function.
9. Try $fgg^{-1}(x)$.

4D Inverse functions, p95

Key point 4.6 is a slight simplification, as it is only true if x is in the domain of both $f(x)$ and $f^{-1}(x)$.

For example, $\sqrt{x^2} = x$ only if $x \geq 0$. However, this is not required for the IB syllabus.

Hints for the grade 7 questions:

11. Set $gg(x) \equiv x$, multiply out the fractions and compare coefficients.

4E Rational functions, p101

The 'Theory of knowledge issues' box on p104 is intended to get the students to think about what mathematical knowledge is convention and what is needed for consistency.